Axion monodromy and inflation

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arxiv:1101.0026 with Nemanja Kaloper (UC Davis), AL, and Lorenzo Sorbo (U Mass Amherst)

Silverstein and Westphal, arxiv:0803.3085
McAllister, Silverstein, and Westphal, arxiv:0808.0706
+ others

Related work in progress with Sergei Dubovsky (NYU), AL, and Matthew Roberts (NYU)
and with Dubovsky, Raphael Flauger (Yale), Kaloper, and AL

I. Introduction: "high scale inflation" in UV-complete theories

II. Monodromy inflation in string theory

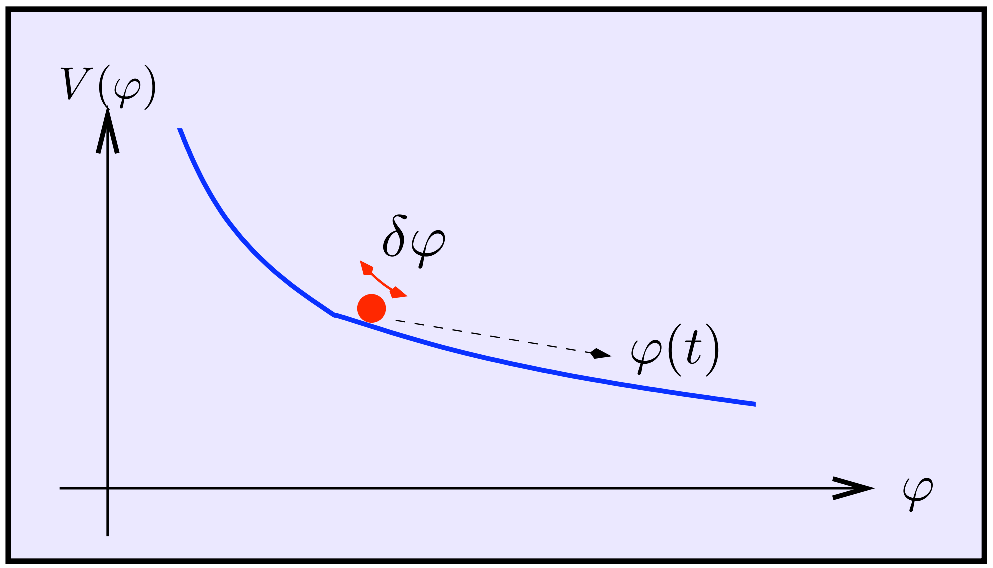
III. 4d models of axion monodromy

IV. Quantum corrections

V. Conclusions

I. Introduction

"Slow roll" inflation matches CMB/LSS data well



Slow roll + vacuum dominance:

- $\epsilon = m_p^2 \left(\frac{V'}{V}\right)^2 \ll 1$
- $\eta = m_p^2 \frac{V''}{V} \ll 1$

Spacetime approximately de Sitter: $ds^2 = -dt^2 + e^{2\int dt H} d\vec{x}_3^2 \ , \quad H^2 = \frac{V}{m_p^2}$

Observed flatness of space: φ must roll long enough for space to expand by e^{60}

Quantum fluctuations generate structure: $\frac{\delta\rho}{\rho} \sim \frac{V^{3/2}}{m_p^3 V'} \sim \frac{\delta T_{CMB}}{T_{CMB}}$

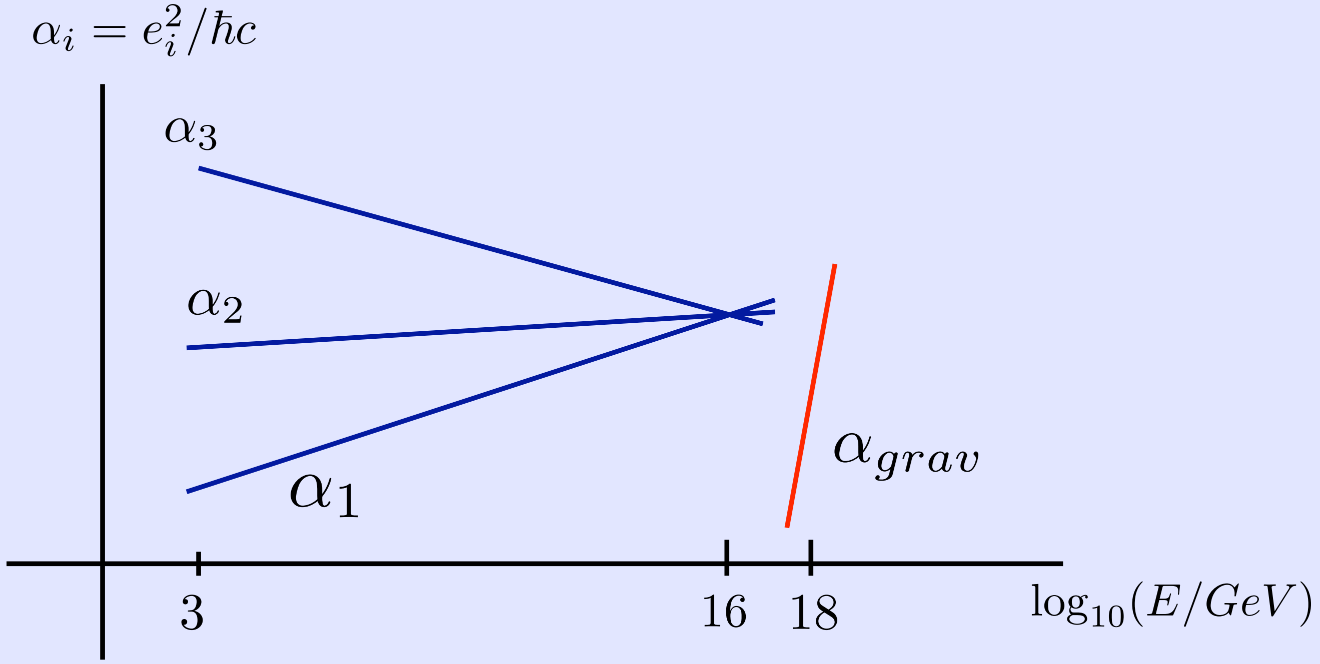
Quantum fluctuations generate gravitational waves: $\mathcal{P}_g \sim \frac{V}{m_p^4}$

Detectable via CMB polarization experiments

Scale of inflation

Observational upper bound on GW: $V \lesssim 10^{16}\ GeV \sim M_{GUT}$

Close to "unification scale"



Couplings unify (assuming MSSM above 1 TeV) at approximately 10^{16} GeV. Graph not to scale.

See also:

- p decay
- ν mass

If V near upper bound: detectable by PLANCK or ground-based CMB polarization experiments

Detectable gravitational radiation requires large fields

Lyth, hep-ph/9606387

- $\left(\frac{\delta\rho}{\rho}\right) = c\frac{V^{1/2}}{m_{pl}^3}\frac{V}{V'} \sim 10^{-5}$ from observations

upper bound on V $\Rightarrow$ upper bound on V'/V

- $N_e = \int dt H = \int \frac{d\phi}{\dot{\phi}} H = \frac{3}{m_{pl}^2}\int d\phi \frac{V}{V'} \gtrsim 60$

to match observed flatness

upper bound on $\frac{d\phi}{dN}, \Delta\phi$ during inflation

$$\Rightarrow \Delta\varphi \gg m_{pl}$$

Effective field theory and large ϕ

Effective field theory: expansion in 1/M for some UV scale M

$$V = \sum_n g_n \frac{\phi^n}{M^{n-4}}$$

generically

- $g_n \sim 1$ unless forbidden by symmetry
- $M \lesssim m_{pl}$

Expansion breaks down for $\phi > M$

- New degrees of freedom could become light
- Relevant d.o.f. very different

Inflation is a highly nongeneric theory

Consider $V \sim m^2\phi^2$ or $V \sim \lambda\phi^4$

$$\delta\rho/\rho \sim 10^{-5}, \; N_e \gtrsim 60 \quad \Rightarrow$$

- $\frac{m^2}{m_{pl}^2} \sim 10^{-12}$
- $\lambda \sim 10^{-14}$

Corrections $\delta V = \sum_n g_n \frac{\phi^n}{M^{n-4}}$

all g_n must be small: infinite fine tuning!

else e.g. $\eta = m_{pl}^2 \frac{V''}{V} \geq 1$

Slow roll inflation requires approximate shift symmetry

$$\phi \to \phi + a$$

Perturbative quantum corrections

Small couplings $\frac{m^2}{m_{pl}^2}, \ \lambda$

m_{pl} -suppressed couplings to gravity

$\Rightarrow$ loops of inflaton, graviton gives suppressed couplings

$$V_{loop} = V_{class} F\left(\frac{V}{m_{pl}^4}, \frac{V'}{m_{pl}^2}, \ldots\right)$$

Coleman and Weinberg; Smolin; Linde

Slow roll inflation safe against inflaton, graviton loops

perturbative corrections preserve symmetries

UV completions make slow roll difficult to maintain

Continuous global symmetries like $\phi \to \phi + a$ are always (we think) broken

- Gravity breaks continuous global symmetries (Hawking radiation/virtual black holes, wormholes,...)

Holman et al; Kamionkowski and March-Russell; Barr and Seckel; Lusignoli and Roncadelli; Kallosh, Linde, and Susskind

- String theory: continuous global symmetries tend to be gauged, anomalous

- Anomalous symmetries broken by nonperturbative effects (e.g. Peccei-Quinn symmetry of axion)

$$\delta V \sim \Lambda^4 \sum_n c_n \cos(n\phi/f_\phi)$$

One attempt: "pseudonatural inflation"

Use anomalous symmetry to generate potential

$$V = \Lambda^4 \cos\left(\frac{\phi}{f_\phi}\right) + \dots$$

$$\delta V \sim \Lambda^4 \sum_{n>1} c_n \cos(n\phi/f_\phi)$$

$$c_n \sim e^{-nS}, \quad \left(\frac{f_\phi}{M}\right)^n$$

Λ some dynamical scale; slow roll for $f_\phi \gg \Lambda$

large field if $f_\phi \gg m_{pl}$

Problem: $f_\phi > m_{pl}$ with c_n small does not seem to be allowed

$$\frac{f_\phi}{M} \gg 1$$

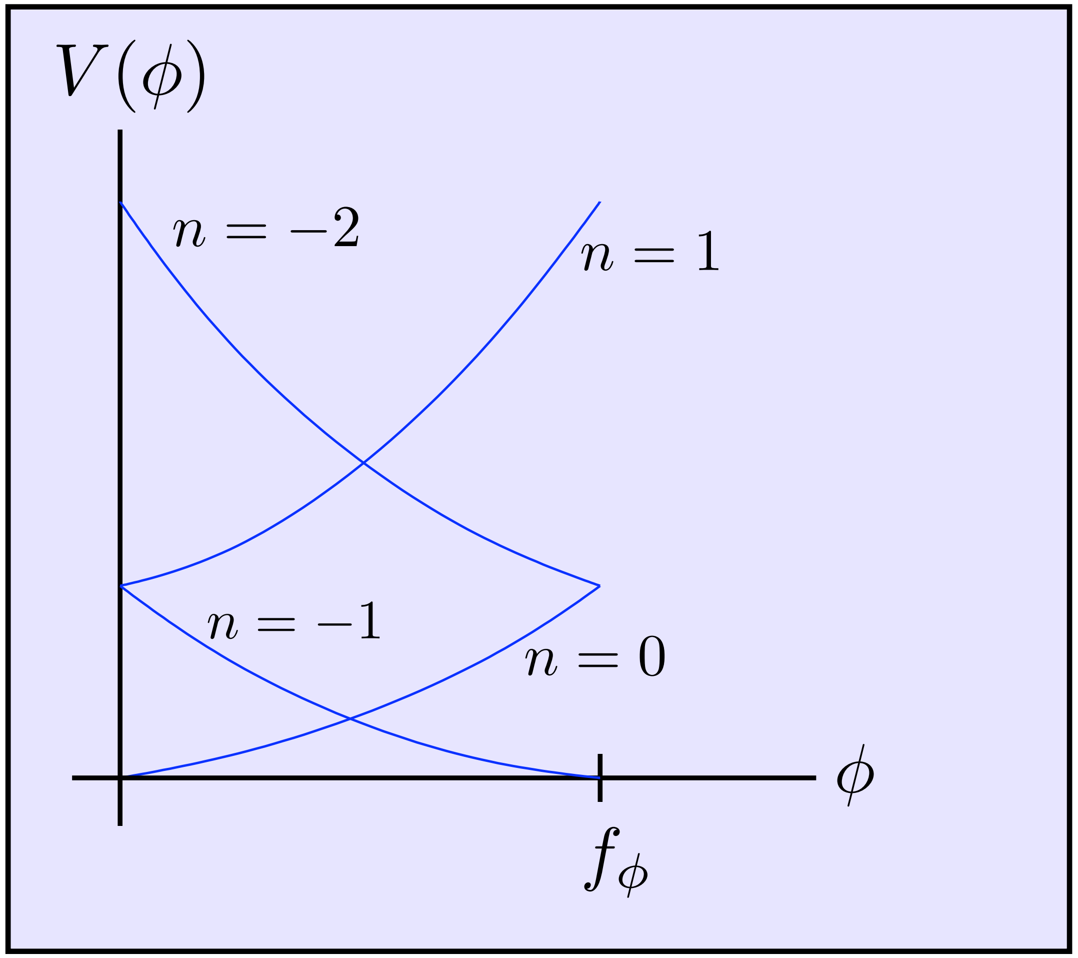
Banks, Dine, Fox, and Gorbatov;
Arkani-Hamed, Motl, Nicolis, and Vafa

II. Monodromy inflation in string theory

Consider compact scalar field $\varphi \sim \varphi + f \; ; \; f \ll m_{pl}$

Silverstein and Westphal; McAllister, Silverstein, and Westphal

Theory invariant under shift $\varphi \to \varphi + f$ *physical state* need not be



Let axion wind N times such that $N f_\phi \gg m_{pl}$

Compactness of field space seems to control quantum corrections

Cartoon

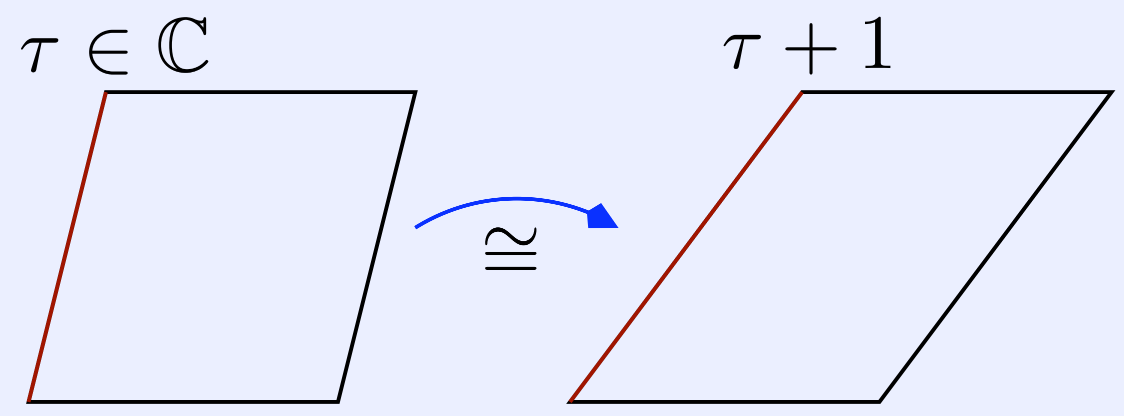
Most models to date constructed within string theory

But see Berg, Pajer, and Sors; Kaloper and Sorbo

Illustrative example: type IIA with D4-brane wrapped on 2-torus

- τ has period =1
- $\phi = m_{pl}\tau$

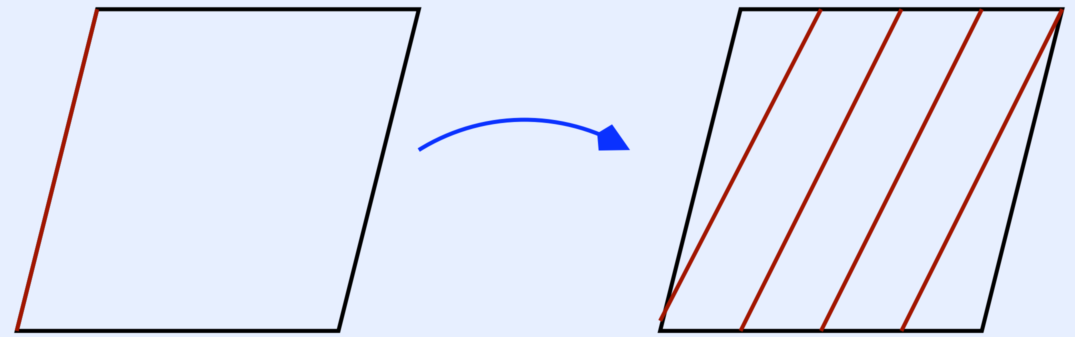
  canonically normalized scalar



Shift $\tau \to \tau + 1$ is symmetry of torus, but stretches D-brane.

$$V(\phi) \sim \frac{m_s^4}{g_s}\sqrt{1+(m_{pl}\phi)^2}$$

n = # of D4 windings



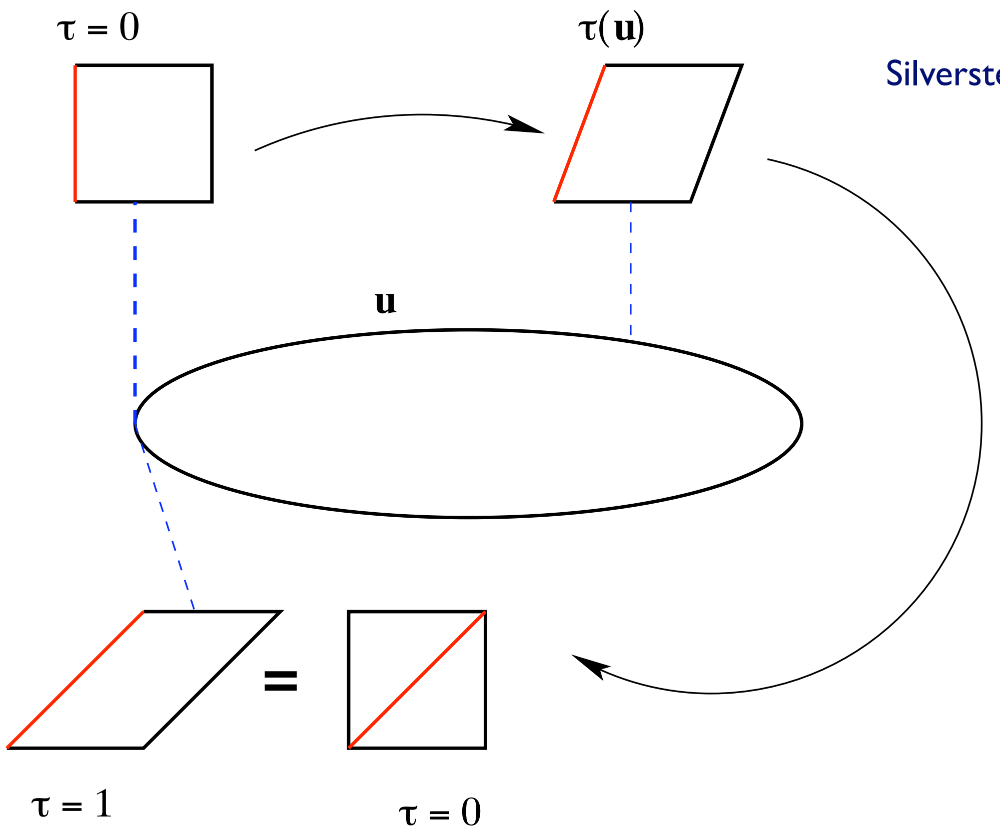
Shift τ n times; D-brane becomes n times as long.

Doesn't quite work but illustrates point

Example: nilmanifold N

T^2 fibration of S^1 with monodromy $\tau \to \tau + M$

Red: D4-brane wrapped on single cycle of torus



Silverstein and Westphal

Consider $M_6 = (\mathrm{Nil})_1 \times (\mathrm{Nil})_2$

Wrap D4 on linear combinations of $S^1_{1,2} \subset T^2_{1,2}$

$$S_{DBI} \sim -\int d^4x M^4 \sqrt{(A + Bu^2)(1 - C(\partial u)^2)}$$

Inflation works at large u:

$$S \sim au(\partial u)^2 - bu \sim (\partial\phi)^2 - \mu^{10/3}\phi^{2/3}$$

Example: axion monodromy

McAllister, Silverstein, and Westphal

$$S^2 \subset M_6 \; ; \; a = \int_{S_2} C^{(2)}_{RR} \qquad\qquad a \equiv a + 2\pi$$

$$\mathcal{L} = \tfrac{1}{2} f_a^2 (\partial a)^2 = \tfrac{1}{2}(\partial\phi)^2$$

Now add NS5-brane wrapping S^2

$a \longrightarrow a + 2\pi$ induces D3-brane charge

$$V \sim \tfrac{1}{g_s^2}\sqrt{C + Da^2}$$

Again, slow roll inflation works only at large a

$$\mathcal{L} \sim \tfrac{1}{2}(\partial\phi)^2 - \mu^3\phi$$

- Known string realizations seem to give flat potentials, with relatively small powers $V \sim M^{4-p} \varphi^{p<2}$

  Seems to the result of coupling to moduli, KK modes

  Dong, Horn, Silverstein, and Westphal

  Is a quadratic potential viable?

  CMB data: p <=2 viable, smaller p more viable

- Quantum corrections studied model by model: these are complicated, and physical reason for flat potentials is not completely transparent.

Effective field theory approach

- Input basic fields, symmetries, topology of field space
- Expand action in powers of 1/M (M = UV scale), include all terms consistent with symmetries
- Pinpoints physics behind suppressing corrections to slow roll
- Isolates fine tuning required.
- Provides a framework for building new string models

String theory has a complicated landscape
Realistic models very hard to construct
Quantum corrections difficult to compute

$\Rightarrow$ 4d effective field theory analysis is *always* important

III. 4d models of axion monodromy

Axion-four form model

Kaloper and Sorbo

$$S_{class} = \int d^4x \sqrt{g} \left(m_{pl}^2 R - \frac{1}{48} F^2 - \frac{1}{2} (\partial\varphi)^2 + \frac{\mu}{24} \varphi^* F \right)$$

$F_{\mu\nu\lambda\rho} = \partial_{[\mu} A_{\nu\lambda\rho]}$ U(1) gauge symmetry: $\delta A_{\mu\nu\lambda} = \partial_{[\mu} \Lambda_{\nu\lambda]}$

φ periodic: $\varphi \to \varphi + f_\varphi$

F does not propagate.
U(1) quantized $F_{\mu\nu\lambda\rho} = n e^2 \epsilon_{\mu\nu\lambda\rho}\ ;\ n \in \mathbb{Z}$

n can jump across domain walls/membranes

Dynamics

Single massive scalar degree of freedom   Dvali; Kaloper and Sorbo

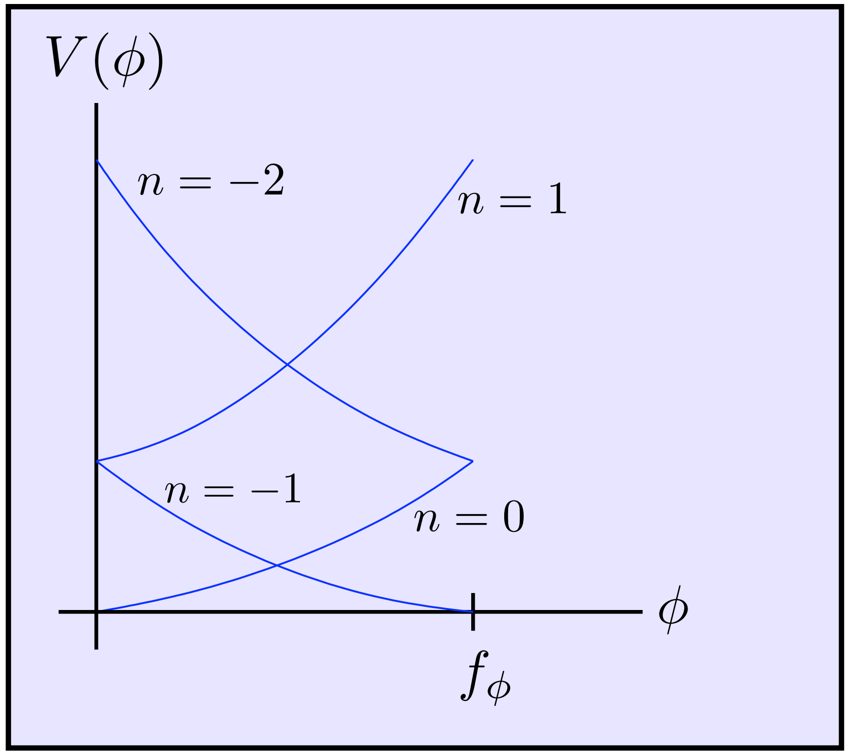
Hamiltonian: $H_{tree} = \frac{1}{2}p_\phi^2 + \frac{1}{2}\left(p_A + \mu\phi\right)^2 + \ grav.$

Compact U(1): $p_A = ne^2$

p_A conserved by H_{tree}

Jumps by membrane nucleation

Consistency condition: $\mu f_\varphi = e^2$



Realizes monodromy inflation: theory invariant if

$$\varphi \to \varphi + f_\varphi \ ; n \to n-1$$

Good model for inflation: fits data well if $\mu \sim 10^{-6} m_{pl}$

+ observable GW

Large-N gauge dynamics

$$S_{class} = \int d^4x\sqrt{g}\left(m_{pl}^2 R - \frac{1}{4g_{YM}^2}\mathrm{tr}G^2 - \frac{1}{2}(\partial\varphi)^2 + \frac{\varphi}{f_\varphi}\mathrm{tr}G\wedge G\right)$$

G: field strength for U(N) gauge theory with N large; strong coupling in IR

Instanton expansion breaks down

Witten; Giusti, Petrarca, and Taglienti

$$H_{tree} = H_{gauge} + \frac{1}{2}p_\varphi^2 + \frac{1}{2}\left(n\Lambda^2 + \mu\varphi\right)^2$$

Λ strong coupling scale of U(N) theory

$$\mu = \Lambda^2/f_\varphi$$

Can be related to 4-form version: $F_{\mu\nu\lambda\rho} \sim \mathrm{tr}\ G_{[\mu\nu}G_{\lambda\rho]}$ Dvali

IV. Quantum corrections

$$S_{class} = \int d^4x \sqrt{g} \left(m_{pl}^2 R - \frac{1}{48} F^2 - \frac{1}{2} (\partial \varphi)^2 + \frac{\mu}{24} \varphi^* F \right)$$

$\mu \sim 10^{-6} m_{pl}$ to match constraints on $\delta \rho / \rho, \; N_e$

What are the possible corrections?

Effective field theory:

- Allow all terms consistent with symmetries, topology of field space
- Dimenson-d operators suppressed by M_{uv}^{d-4}

Corrections controlled by:

- Compactness of scalar, U(1)
- Small coupling $\mu / M_{uv} \ll 1$

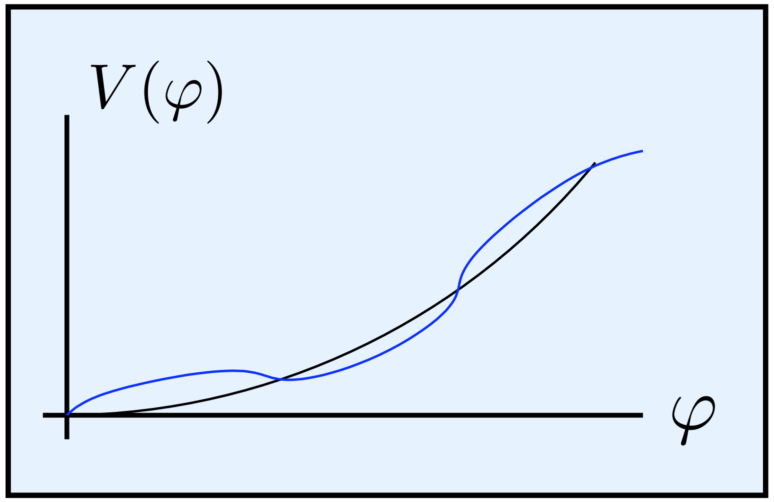
Direct corrections to $V(\varphi)$

Periodicity of φ $\Rightarrow$ quantum corrections to S must be

- Functions of $\partial^n \varphi$
- periodic functions of φ

$$\delta V \sim \Lambda^4 \sum_{n>1} c_n \cos(n\varphi/f_\varphi)$$

$$f_\phi \ll m_{pl}$$



Monodromy potential modulated by periodic effects

$$V_{corr} \ll \frac{1}{2}\mu^2\varphi^2 \Rightarrow \Lambda^4 \ll M_{gut}^4$$

$$\eta = m_{pl}^2 \frac{V''}{V} \ll 1 \Rightarrow \frac{\Lambda^4}{f_\varphi^2} \ll \frac{V}{m_{pl}^2} = H^2$$

Example: feasible if $\Lambda \sim .1\ M_{gut},\ f > .01\ m_{pl}$

- Gauge dynamics: $\Lambda = \Lambda_{QCD}$

  from couplings $\frac{\varphi}{f_\varphi} \mathrm{tr}\, G \wedge G$

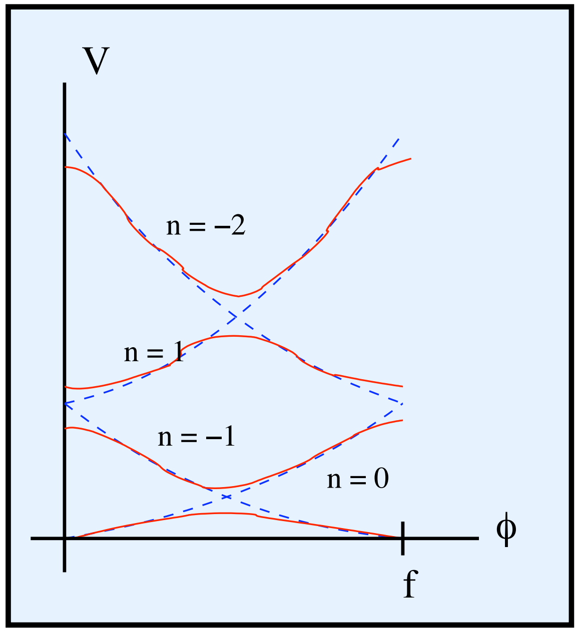
  instanton corrections take above form (if dilute gas approx good)

  strong coupling effects (when dilute gas aprox fails)

$$\delta V \sim \Lambda^4 \min_k F\left(\frac{\varphi}{f_\varphi} + k\right)$$

Witten; Giusti, Petrarca, and Taglienti

multibranched function of φ



When using this effect to generate monodromy potential: mixing between branches must be weak

When this generates corrections: mixing must be strong (else trapped in a fixed branch)

- Gravitational dynamics: $\Lambda^4 \sim \frac{f_\varphi^{n+4}}{m_{pl}^n}$

  gravitational instantons, wormholes, etc.

Caveat: moduli stabilization

In any string theory: couplings in V will depend on moduli ψ

$$V = V_0(\psi) + \frac{1}{2}\mu^2\left(\frac{\psi}{m_{pl}}\right)\varphi^2 + \Lambda^4 \sum_n c_n\left(\frac{\psi}{m_{pl}}\right)\cos\left(\frac{n\varphi}{f_\varphi}\right)$$

Periodic corrections change sign many times since $f_\phi \ll m_{pl}$

Moduli must be stabilized by different
effects than instantons coupling to inflaton

$$M_\psi^2 \equiv V_0''(\psi) \gg \frac{\Lambda^4}{m_{pl}^2}$$

Large $\varphi \gg m_{pl}$ sources potential for ψ

Stability requires $M_\psi^2 \gg \mu^2\varphi^2/m_{pl}^2 \sim \mu^2/\epsilon \sim H^2$

Indirect corrections to $V(\varphi)$

Additional corrections must respect periodicity of φ

$\Rightarrow$ corrections to dynamics of four-form F

$$S_{class} = \int d^4x \sqrt{g} \left(m_{pl}^2 R - \frac{1}{48} F^2 - \frac{1}{2} (\partial\varphi)^2 + \frac{\mu}{24} \varphi^* F \right)$$

Consider $\delta\mathcal{L} = \sum_n d_n \frac{F^{2n}}{M^{4n-4}}$

Integrate out F: $F \sim \mu\varphi + \ldots$

$$\delta V_{eff} = V_{class} \times \left(\sum_{n=1} d_{n+1} \frac{V_{class}^n}{M^{4n}} \right)$$

Safe if: $M^4 \gg V_{class} \sim M_{gut}^4$

Corrections of the form $\delta\mathcal{L} = \left(\sum_{n=1} d_{n+1} \frac{F^{2n}}{M^{4n}} \right) (\partial\varphi)^2$

Gives same effect after redefining φ to be canonically normalized

Small M not always fatal

Many string theory scenarios:

$$V(\varphi) = M_1^4 \sqrt{1 + \frac{\varphi^2}{M_2^2}} \qquad M_2 \ll m_{pl}$$

Silverstein and Westphal;
McAllister, Silverstein, and Westphal

- For small φ $\quad V \sim \frac{1}{2}\mu^2\varphi^2 \;; \mu = \frac{M_1^4}{M_2^2}$
- For $\varphi \gg m_{pl}$ $\quad V \sim m^3\varphi; \; m^3 = \frac{M_1^4}{M_2}$

Out of range of 4d effective field theory; requires understanding of UV completion (eg 10d SUGRA) to compute

Example: backreaction on compactification

Consider string modulus ψ

determines KK scale: $L_0 e^{-\psi/m_{pl}}; \mathcal{V}_D \sim L^D;\ m_{pl}^2 = m_*^{D+2}\mathcal{V}_D$

$$\mathcal{L}_\psi = \frac{1}{2}(\partial\psi)^2 - \frac{1}{2}M_\psi^2\psi^2 + c\frac{\psi}{m_{pl}}F^2 + \dots$$

Integrate out ψ : $\frac{\psi}{m_{pl}} = c\frac{F^2}{M_\psi^2 m_{pl}^2} \sim c\frac{V}{m_{pl}^2 M_\psi^2} = c\frac{H^2}{M_\psi^2}$

$$\frac{\delta m_{pl}^2}{m_{pl}^2} \sim \frac{H^2}{M_\psi^2}$$

Since $\eta = m_{pl}^2\frac{V''}{V}\ ; \epsilon = m_{pl}^2\frac{(V')^2}{V^2}$

We must have $\frac{\delta m_{pl}^2}{m_{pl}^2} \sim \frac{H^2}{M_\psi^2} \ll 1$ Moduli coupling to inflaton must be fairly heavy

If coupling to F is: $\sim \frac{(\psi-\psi_0)^2}{m_{pl}^2}F^2$ corrections proportional to $\frac{\psi_0}{m_{pl}}$ Dong, Horn, Silverstein, and Westphal

$\frac{\psi_0}{m_{pl}} \sim 1$ also edge of validity of effective field theory

Example: Coleman-Weinberg corrections

Consider scalar fields ψ_n (e.g. moduli, KK states, etc.)

$$\delta\mathcal{L} \sim \frac{1}{2}(\partial\psi_n)^2 - \frac{1}{2}M_n^2\psi_n^2 - \sum_k d_{n,k}\frac{F^{2n}}{M^{4n-2}}\psi_n^2$$

Integrate out F: $F^2 \sim V_{class} = \frac{1}{2}\mu^2\varphi^2$

Effective mass for ψ : $M_{eff}^2 = M_\psi^2 + M^2\sum_k d'_{n,k}\frac{V^2}{M^{4n}}$

Integrate out ψ_n : $\delta V_{CW}(\varphi) \sim M_{eff}(\phi)^4 \, \ln\frac{M_{eff}}{M}$

Must include all such states with $M_n^2 < M^2$

Corrections safe if $n_{eff}M_\psi^2 \ll M^2 \; ; V \ll M^4$

Kaluza-Klein corrections

Roughly $n_{eff} = \frac{m_{pl}^2}{m_*^2}$; $m_* = (m_s, m_{pl,10}) \gtrsim M_{gut}$

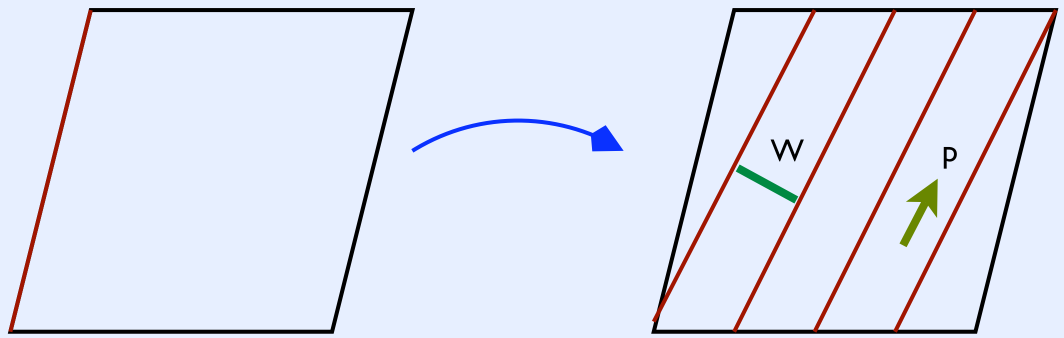
$$
\begin{aligned}
V_{CW} &= \sum_{KK} \int d^4q \, \ln\left(q^2 + M_{n,eff}^2\right) \\
&\sim \mathcal{V}_D \int d^{D+4}q \, \ln \left(q^2 + \sum_k d_k \frac{V_{tree}^k}{M^{4k-4} m_{pl}^2} \right) \\
&\sim m_*^{D+4} \mathcal{V}_D(\psi) + m_*^2 \mathcal{V}_\mathcal{D} \sum_k d_k \frac{V_{tree}^k}{M^{4k-4} m_{pl}^2} \\
&\sim \delta V(\psi) + V_{tree} \, F\left(\frac{V_{tree}}{M^4}\right)
\end{aligned}
$$

Corrections safe if $V_{class} \ll M^4$

NB if KK mode couples to F as $\frac{(\psi_n - \psi_{0,n})^2}{m_{pl}^2} F^2$ tree level corrections subleading if

$H^2 < M_{KK}^2$; $\psi_{n,0} < m_{pl,10}$

Additional "stringy" light states



Shift τ n times; D-brane becomes n times as long.

Consider square torus with sides of length L; D4 wrapped n times

$$m_W^2 = \frac{m_s^4 L^2}{1+n^2};\ m_p^2 = \frac{1}{L(1+n^2)}\ ; n = \frac{\varphi}{f_\varphi} = \frac{F}{\mu f_\varphi}$$

n >> 1: strings have spectrum of asymmetric torus with sides of length

$$L_W = \frac{n}{m_s^2 L}\ ; L_p \sim \frac{n}{L}$$

and volume $V_{eff} \sim \frac{n^2}{m_s^2} \sim \frac{F^2}{m_s^2 e^4}$

where $e^2 = \mu f_\varphi$ is unit of quantization of F flux

Leading quantum correction

$$
\begin{aligned}
V_{CW} &= \sum_{k,l} \int d^4q \ \ln\ \left(q^2 + m^2_{W,k} + m^2_{p,k}\right) + \dots \\
&\sim \frac{F^2}{m_s^2 e^4} \int d^6q \ \ln q^2 + \dots \\
&\sim \frac{m_s^4}{e^4} F^2 + \dots
\end{aligned}
$$

Effect is to renormalize $e^2 \rightarrow m_s^2 \sim M^2_{gut} \sim 10^{-4} m^2_{pl}$

Dangerous: $\mu = 10^{-6} m_{pl}$ to match observation

$$\Rightarrow \quad f_\varphi \sim 10^2 \ m_{pl}$$

Must ensure renormalization of e is suppressed:

$$f_\varphi \sim .1 \ m_{pl} \Rightarrow \quad e^2 \sim (.1 M_{gut})^2$$

• NB model above is crude (and known not to work for other reasons) so this is a caveat and not a fatal flaw

• Even if μ^2 pushed above $10^{-6} m_{pl}$

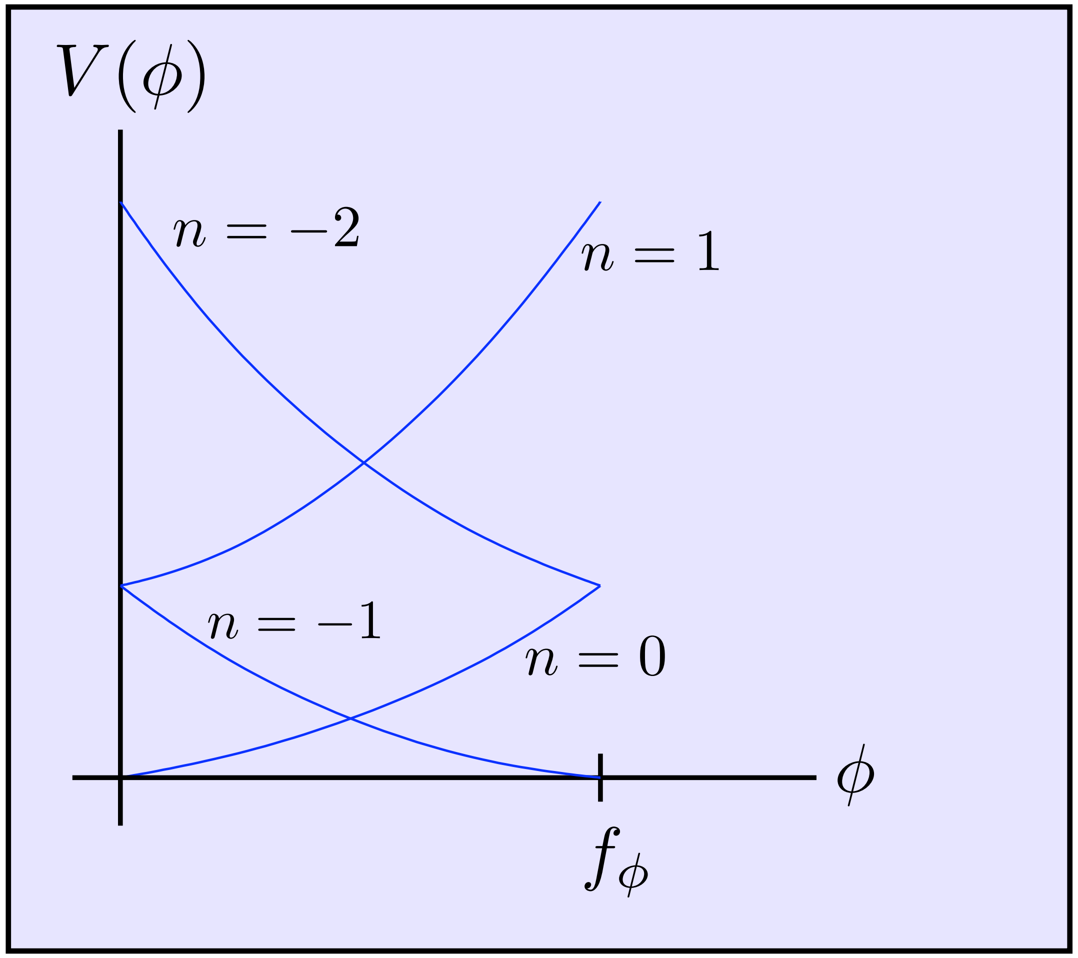
we may still get successful large field inflation of the form, e.g.

$$V(\varphi) = M_1^4 \sqrt{1 + \frac{\varphi^2}{M_2^2}}$$

but this requires more than our 4d EFT can do at present

Stability

Sergei Dubovsky, AL, Matthew Roberts;
SD, AL, Raphael Flauger, in progress



Success of monodromy inflation requires that transition between branches is slow compared to time scale of inflation (must complete 60 efolds before such transitions)

Bounds on membrane tension

Transitions occur by bubble nucleation. Let:

- T = tension of bubble wall
- E = energy difference between branches

Decay probability: $\Gamma \sim \exp\left(-\frac{27\pi^2}{2}\frac{T^4}{E^3}\right)$ (thin wall) Coleman

Phenomenological bound on T

$$\varphi = N f_\varphi \; ; \Delta\varphi = f_\varphi$$

$$E \sim \Delta V \sim V'(\varphi) f_\varphi \sim \frac{V}{N}$$

$$\Gamma \ll 1 \Rightarrow T^{1/3} \gg \left(\frac{2}{27\pi^2 N^3}\right)^{1/4} V^{1/4}$$

Let: $f_\phi \sim .1\ m_{pl};\ N \sim 100; V \sim M_{gut}^4$

$$T \gg (.2V^3)^{1/4} \sim (.9M_{gut})^3$$

Borderline; should check against explicit models

V. Conclusions

- Check stability in explicit string, field theory models

Dubovsky, AL, Roberts; SD, Flauger, NK, AL, in progress

- General issue: monodromy inflation does not seem *parametrically* safe. Should we worry?

Perhaps this is interesting:

- Implies number of e-foldings could be close to lower bound
- Implications for measurements of curvature, pre-inflation transients

- Other interesting applications of axion monodromy

Kerr black holes; axion condensation via Penrose process. Instability can lead to observable axion decays

Dubovsky and Gorbenko